Scattering Solution of Three-Dimensional Array of Patches Using the Recursive T-Matrix Algorithms

Levent Gürel, *Member, IEEE*, and Weng Cho Chew, *Senior Member, IEEE*

***Abstract*—The recursive T-matrix algorithms are used to solve for the vector electromagnetic scattering problem of a three-dimensional array of patches. The formulation uses only the E_x and E_y components of the electromagnetic field wherein the three-dimensional scalar addition theorem can be used. The coefficients for the scalar addition theorem is calculated with an efficient recurrence relation. This results in reduced memory requirement and computation time. When the addition theorem is violated, a generalized recursive T-matrix algorithm is used to mitigate the problem caused by the violation of the addition theorem. The scattering solutions are validated by comparison with the method of moments and the reduced computational complexity of the solution is demonstrated.**

I. Introduction

Recently, we have developed recursive T-matrix algorithms (RTMA's) using translation formulas [1]–[6]. These algorithms have reduced computational complexity compared to conventional solution techniques. They have been demonstrated to work efficiently for calculating two-dimensional scattering solutions for both transverse-magnetic (TM) and transverse-electric (TE) polarizations. However, they were not demonstrated for three-dimensional vector electromagnetic scattering problems.

In this letter, we show for the first time the use of such algorithms to solve three-dimensional vector electromagnetic scattering problems involving three-dimensional scatterers [7]. To save computer memory, the problem is formulated using only the E_x and E_y components of the electromagnetic field. In implementing the recursive algorithms in three dimensions, the corresponding addition theorem is needed. Since E_x and E_y satisfy the scalar wave equation, the scalar addition theorem can be used. Recently, we have developed a recurrence relation to efficiently calculate the coefficients of the scalar addition theorem in three-dimensions [8]. Such recurrence relations will be used in the calculation of the scalar addition theorem needed for the translation formulas.

Manuscript received December 12, 1991. This work was supported by the National Science Foundation under grant NSF ECS-85-25981 and the Office of Naval Research under Grant N000-14-89-J-1286, with a matching funds from Northrop and General Electric. The computer resources were provided by the National Center for Supercomputing Applications, Urbana, Illinois, and Cray Research, Inc.

L. Gürel was with the Electromagnetics Laboratory, Department of Electrical and Computer Engineering, University of Illinois, Urbana, IL 61801. He is now with the IBM Research Division, T.J. Watson Research Center, P.O. Box 218, Yorktown Heights, NY 10598.

W. C. Chew is with the Electromagnetics Laboratory, Department of Electrical and Computer Engineering, University of Illinois, Urbana, IL 61801.

IEEE Log Number 9108193.

The solutions are validated by comparison with the method of moments (MOM) [9] and the computation times are compared. The recursive algorithms are shown to have reduced computational complexity.

II. Formulation

Since the x and y components of the electric field will be used to represent the electromagnetic fields, following the standard T-matrix notation [5]–[7], the incident field can be expressed as

$$\boldsymbol{E}_t^I(\boldsymbol{r}) = \begin{bmatrix} E_x^I(\boldsymbol{r}) \\ E_y^I(\boldsymbol{r}) \end{bmatrix} = \begin{bmatrix} \Re g\psi_x^t(\boldsymbol{r}) & 0 \\ 0 & \Re g\psi_y^t(\boldsymbol{r}) \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{e}_x \\ \boldsymbol{e}_y \end{bmatrix} = \Re g\bar{\boldsymbol{\Psi}}^t(\boldsymbol{r}) \cdot \boldsymbol{e}. \quad (1)$$

When N scatterers are present, the scattered field can be expressed as

$$\boldsymbol{E}_t^S(\boldsymbol{r}) = \begin{bmatrix} E_x^S(\boldsymbol{r}) \\ E_y^S(\boldsymbol{r}) \end{bmatrix} = \sum_{i=1}^{N} \begin{bmatrix} \psi_x^t(\boldsymbol{r}_i) & 0 \\ 0 & \psi_y^t(\boldsymbol{r}_i) \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{f}_{xi} \\ \boldsymbol{f}_{yi} \end{bmatrix} = \sum_{i=1}^{N} \bar{\boldsymbol{\Psi}}^t(\boldsymbol{r}_i) \cdot \boldsymbol{f}_i. \quad (2)$$

In this equation, $\psi(\boldsymbol{r}_i)$ is a column vector containing the scalar spherical harmonics for outgoing waves that involves spherical Hankel functions. The prefix "$\Re g$" implies the "regular part," hence, $\Re g\psi(\boldsymbol{r})$ contains spherical harmonics for standing waves which involve spherical Bessel functions. The position vector $\boldsymbol{r}_i$ originates from the center of the ith scatterer.

Then, T matrices, which relate the scattered-field coefficients to the incident-field coefficients, are defined for each scatterer via the relation

$$\boldsymbol{f}_i = \begin{bmatrix} \boldsymbol{f}_{xi} \\ \boldsymbol{f}_{yi} \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{T}}_{i(N)}^{xx} & \bar{\boldsymbol{T}}_{i(N)}^{xy} \\ \bar{\boldsymbol{T}}_{i(N)}^{yx} & \bar{\boldsymbol{T}}_{i(N)}^{yy} \end{bmatrix} \cdot \begin{bmatrix} \bar{\beta}_{i0}^x & 0 \\ 0 & \bar{\beta}_{i0}^y \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{e}_x \\ \boldsymbol{e}_y \end{bmatrix} = \bar{\boldsymbol{T}}_{i(N)} \cdot \bar{\beta}_{i0} \cdot \boldsymbol{e} \quad (3)$$

so that the scattered field can be expressed in terms of these T matrices, i.e.,

$$\boldsymbol{E}_t^S(\boldsymbol{r}) = \sum_{i=1}^{N} \bar{\boldsymbol{\Psi}}^t(\boldsymbol{r}_i) \cdot \bar{\boldsymbol{T}}_{i(N)} \cdot \bar{\beta}_{i0} \cdot \boldsymbol{e}. \quad (4)$$

In (3) and (4), one T-matrix is defined for each scatterer, i.e., the subscript i denotes the ith scatterer. The parenthesized N in the subscript is an "environment parameter," which denotes the presence of N scatterers in the geometry when $\bar{\boldsymbol{T}}_{i(N)}$ is

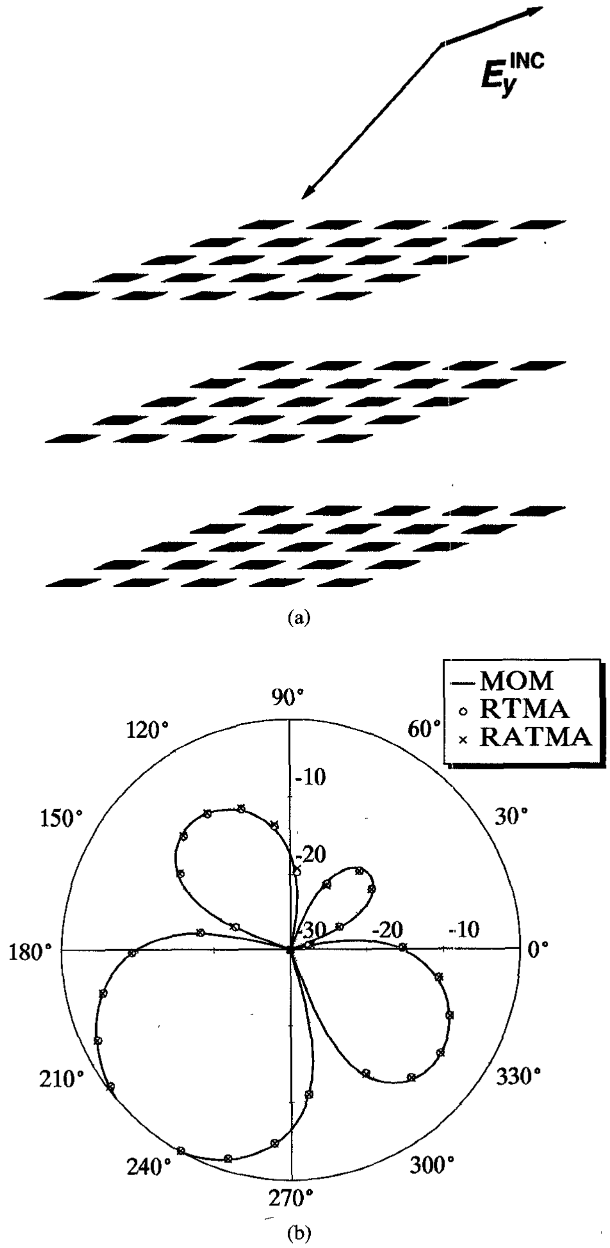

Fig. 1. (a) An E_y-polarized plane wave incident on a three-dimensional clustering of identical square patches with dimension $kw = 1.0$ and spacing $kd = 2.0$. The angles of incidence are $\phi = 0$ and $\theta = 45°$. (b) RCS plots on the $\phi = 0$ cut due to the $3 \times 3 \times 3$ array configuration of three-dimensional clustering of patches as in (a).

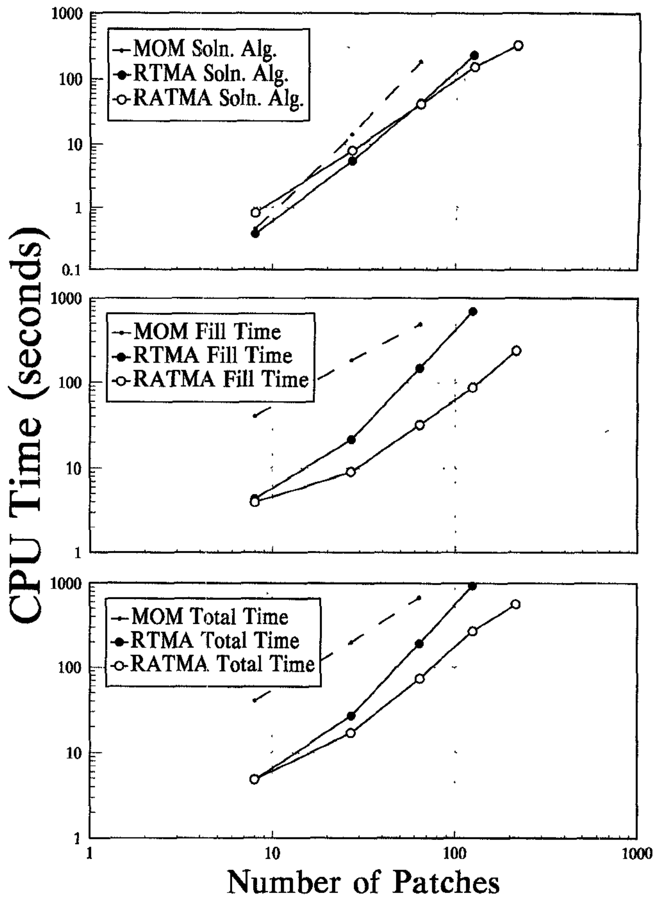

Fig. 2. Comparisons of the computation times required by the applications of the MOM, the RTMA and the RATMA to the three-dimensional clusterings of patches.

defined. The translation matrix $\bar{\beta}_{i0}$ for scalar spherical harmonics is derivable from the scalar addition addition theorem whose elements can be efficiently calculated [8].

A recursive T-matrix algorithm (RTMA) has been previously derived [1], [3] to unravel recursively the N-scatterer solution from a one-scatterer solution. That is, $\bar{\boldsymbol{T}}_{i(N)} \cdot \bar{\beta}_{i0}$ can be found recursively from $\bar{\boldsymbol{T}}_{i(1)}$, the isolated-scatterer T-matrix of one scatterer. The isolated-scatterer T-matrix can be found via the method described earlier [6], [7].

Alternatively, using the addition theorem, (4) can be written as

$$\boldsymbol{E}_t^S(\boldsymbol{r}) = \bar{\boldsymbol{\Psi}}^t(\boldsymbol{r}) \cdot \bar{\tau}_{(N)} \cdot \boldsymbol{e}, \tag{5}$$

where

$$\bar{\tau}_{(N)} = \sum_{i=1}^{N} \bar{\beta}_{0i} \cdot \bar{\boldsymbol{T}}_{i(N)} \cdot \bar{\beta}_{i0} \tag{6}$$

is an aggregate T-matrix for N scatterers. The translation matrix $\bar{\beta}_{0i}$ shifts the coordinate of the ith scatterer to that of a global coordinate whose position vector is $\boldsymbol{r}$. A recursive aggregate-T-matrix algorithm (RATMA) has been derived such that $\bar{\tau}_{(N)}$ can be computed from the one-scatterer solution [4], [5].

When the addition theorem is violated, the corresponding generalized recursive algorithms [10], [11] have also been developed to derive the N-scatterer solution from the one-scatterer solution.

III. Numerical Results

A computer program has been developed using these recursive algorithms for computing the wave scattering solution of a three-dimensional array of patches as shown in Fig. 1(a). These arrays, for instance, have applications in frequency-selective surfaces and artificial dielectrics. However, the goal of this letter is to illustrate that the RTMA's can solve this class of three-dimensional vector problems with reduced computational complexity. For a three-dimensional clustering of the scatterers, the computational complexity of the RTMA is $O(N^{8/3})$ while that for the RATMA is $O(N^{7/3})$ [7]. A direct matrix inversion would have $O(N^3)$ complexity.

Fig. 1(b) shows the radar-cross-section (RCS) plots on the $\phi = 0$ cut (x-z plane) due to a $3 \times 3 \times 3$ array configuration of three-dimensional clustering of patches shown in Fig. 1(a). A plane wave, whose electric field is polarized in the y direction, is incident on the structure at $\phi = 0$ and $\theta = 45°$. The patches are identical and square in shape with size $kw = 1.0$ and spacing $kd = 2.0$ in all of the x, y, and z directions. It is seen that the RTMA's provide solutions that compare very well with those obtained using the MOM.

In Fig. 2, we present the matrix-solution, matrix-fill, and total computation times required by the applications of the MOM, the RTMA, and the RATMA to the three-dimensional clustering of patches shown in Fig. 1. We observe that, for large N, the RATMA has the smallest slope, whereas the MOM has the largest slope in the matrix-solution times, in agreement with the predicted computational complexities of the algorithms. The order of the slopes for the matrix-fill times is just the reverse, i.e., the RATMA has the largest slope, and the MOM has the smallest slope. However, for larger N, the matrix-solution time will be more dominant than the matrix-fill time. Indeed, Fig. 2 shows that the total computation time of the RATMA is starting to be dominated by the matrix-solution time at the last data point ($N = 200$), whereas the total times of the MOM and RTMA are still dominated by the matrix-fill time.

IV. Conclusion

The recursive T-matrix algorithms have been shown to be applicable to three-dimensional vector electromagnetic scattering problem. The RTMA's are shown to agree well with the MOM and, furthermore, they have reduced computational complexities compared to the MOM followed by a Gaussian elimination. Unlike the conjugate-gradient method, the RTMA's provide solutions that are valid for all angles of incidence.

References

[1] W. C. Chew, "An N^2 algorithm for the multiple scattering solution of N scatterers," *Microwave Opt. Technol. Lett.*, vol. 2, no. 11, pp. 380–383, Nov. 1989.

[2] W. C. Chew, J. Friedrich, and R. Geiger, "A multiple scattering solution for the effective permittivity of a sphere mixture," *IEEE Trans. Geosci. Remote Sensing*, vol. 28, no. 2, pp. 207–214, Mar. 1990.

[3] Y. M. Wang and W. C. Chew, "An efficient algorithm for solution of a scattering problem," *Microwave Opt. Technol. Lett.*, vol. 3, no. 3, pp. 102–106, Mar. 1990.

[4] W. C. Chew and Y. M. Wang, "A fast algorithm for solution of a scattering problem using a recursive aggregate tau matrix method," *Microwave Opt. Technol. Lett.*, vol. 3, no. 5, pp. 164–169, May 1990.

[5] W. C. Chew, *Waves and Fields in Inhomogeneous Media.* New York: Van Nostrand Reinhold, 1990.

[6] L. Gürel and W. C. Chew, "A recursive T-matrix algorithm for strips and patches," *Radio Sci.*, accepted for publication, Nov. 1991.

[7] L. Gürel, Recursive algorithms for computational electromagnetics, Ph.D. dissert., Univ. of Illinois, Champaign-Urbana, IL, 1991.

[8] W. C. Chew, "Recurrence relations for three-dimensional scalar addition theorem," *J. Electromagn. Waves Applicat.*, scheduled for publication, 1992.

[9] R. Harrington, *Field Computation by Moment Methods.* New York: Macmillan, 1968. (Reprinted by Krieger, Malabar, FL, 1983.)

[10] W. C. Chew, L. Gürel, Y. M. Wang, G. Otto, R. Wagner, and Q. H. Liu, "A generalized recursive algorithm for wave-scattering solutions in two dimensions," *IEEE Trans. Microwave Theory Tech.*, scheduled for publication, May 1992.

[11] W. C. Chew, Y. M. Wang, L. Gürel, and J. H. Lin, "Recursive algorithms to reduce the computational complexity of scattering problems," in *Proc. 7th Ann. Rev. Prog. Appl. Comp. Electromagn.*, Naval Postgraduate School, Monterey, CA, Mar. 18–22, 1991, pp. 278–291.